ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව

இலங்கைப் பரீட்சைத் திணைக்களம்

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විෂය අංකය / பாட இலக்கம் **01**

විෂයය / பாடம் **Physics**

ලකුණු දීමේ පටිපාටිය / புள்ளி வழங்கும் திட்டம்

I පත්‍රය / பத்திரம் I

ප්‍රශ්න අංකය வினா இல.	පිළිතුරු අංකය விடை இல.	ප්‍රශ්න අංකය வினா இல.	පිළිතුරු අංකය விடை இல.	ප්‍රශ්න අංකය வினா இல.	පිළිතුරු අංකය விடை இல.	ප්‍රශ්න අංකය வினா இல.	පිළිතුරු අංකය விடை இல.	ප්‍රශ්න අංකය வினா இல.	පිළිතුරු අංකය விடை இல.
01.	**05**	11.	**02**	21.	**03/04**	31.	**04**	41.	**03**
02.	**01**	12.	**04**	22.	**02**	32.	**02**	42.	**01**
03.	**05**	13.	**05**	23.	**02**	33.	**02**	43.	**01**
04.	**01**	14.	**04**	24.	**03**	34.	**03**	44.	**01**
05.	**05**	15.	**04**	25.	**02**	35.	**03**	45.	**04**
06.	**03**	16.	**01**	26.	**01**	36.	**04**	46.	**01**
07.	**03**	17.	**05**	27.	**01**	37.	**02**	47.	**04**
08.	**02**	18.	**02**	28.	**03**	38.	**03**	48.	**02**
09.	**03**	19.	**04**	29.	**02**	39.	**05**	49.	**04**
10.	**05**	20.	**03**	30.	**05**	40.	**01**	50.	**05**

PART A – Structured Essay

Answer all four questions on this paper itself.

($g = 10\,m\,s^{-2}$)

1. You are asked to determine the density of the material of a thin uniform wire of length about 15 cm and mass about 200 mg. You are provided with the micrometer screw gauge shown in figure (1) to measure the diameter of the wire.

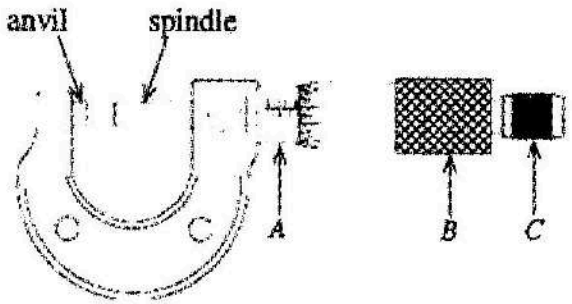


Figure (1)

(*a*) Name the **parts** of the micrometer screw gauge marked as *A*, *B* (not the two scales) and *C*.

A: Sleeve .....(01)

B: Thimble ..... (01)

C: Thimble head/Ratchet .....(01)

(Look for the word sleeve only. e.g. if a student has written sleeve scale accept it)

(*b*) The main scale of the micrometer screw gauge is made by dividing 1 mm into two. The circular scale has 50 equal divisions. One complete turn of *B* will either increase or decrease the distance between the anvil and the spindle by a value which is equal to one division on the main scale.

(i) What is the pitch of the micrometer screw gauge in mm?

0.5 mm .....(01)

(Award marks without the mm unit; but if a student has presented the values with other units look for both the correct value and the unit; Apply the same rule for other answers as well)

(ii) What is the least count of the micrometer screw gauge in mm?

0.01 mm .....(01)

(No mark for a fractional value)

(c) Figure (2) shows the position of the circular scale when the anvil and the spindle touch each other. Determine the value of the zero error of the micrometer screw gauge in mm.

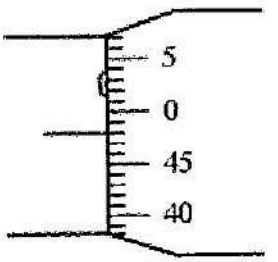


Figure (2)

– 0.02 mm/0.02 mm .....(01)

(d) After determining the zero error, state how the micrometer screw gauge is used to measure the diameter of the wire.

(1) Place the wire in between anvil and spindle by turning *C*/thimble head until *C*/thimble head slips *OR* starts rotating freely *OR* a clicking/ tick tick sound is heard. .....(01)

(2) (Rotate the wire by 90^0 and) measure the diameter at different/several places of the wire. .....(01)

(e) What is the purpose of having part *C* in micrometer screw gauges?

Preventing the spindle from moving further *OR* avoid damaging the wire (measuring object) *OR* to avoid over pressing/pressurizing the wire (measuring object)

.....(01)

(f) (i) Figure (3) shows the position of the circular scale when the diameter of one place of the wire is measured using the micrometer screw gauge mentioned in (c) above.

(1) What is the reading of the micrometer screw gauge in mm?

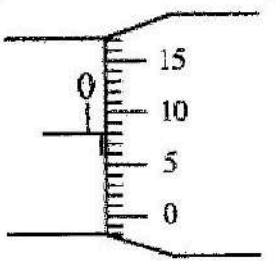


Figure (3)

0.58 mm .....(02)

(2) What is the correct value of the diameter of the wire in mm?

0.60 mm .....(02)

(ii) Calculate the cross-sectional area (in mm^2) of the wire using the value in (*f*) (i) (2) above. (Take $\pi = 3$.)

3×0.3^2 .....(01)

(for substitution with 3 *or* π *or* $\frac{22}{7}$)

0.27 mm^2 .....(01)

(*g*) (i) What other measurements do you take to determine the density of the material of the wire?

(1) Length (of the wire) .....(01)

(2) Mass (of the wire) .....(01)

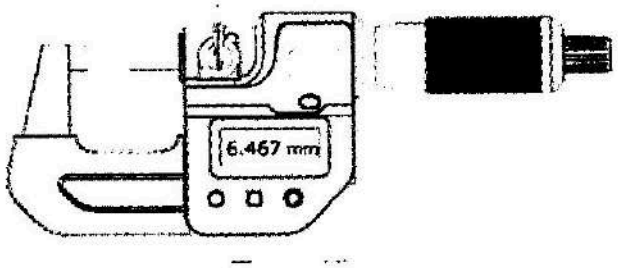
(ii) Name the most appropriate measuring instruments needed to obtain the measurements mentioned in (*g*) (i) above.

(1) Meter ruler .....(01)

(2) Four beam balance *OR* (laboratory) electronic balance *OR* chemical balance .....(01)

(No marks for just stating balance; no marks for triple beam balance since the least count is 100 mg)

(*h*) Figure (4) shows an electronic micrometer screw gauge used in industrial applications. What is the least count of this gauge in mm?



0.001 mm .....(01)

2. You are asked to determine the specific latent heat of vaporization (L) of water using the method of mixtures. Figure (1) shows an incomplete experimental setup. A rubber tube is used to take steam out. A well lagged copper calorimeter, water and a copper stirrer are also provided.

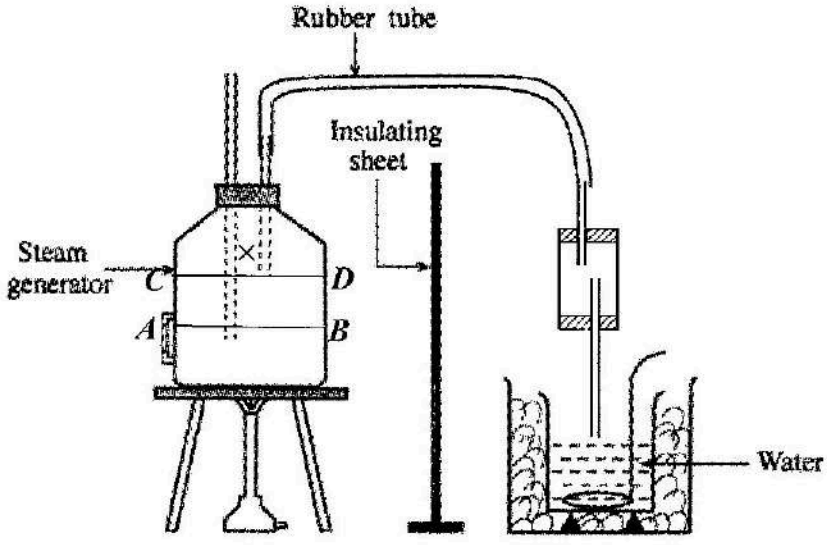


Figure (1)

(*a*) (i) Water has to be poured into the steam generator. Using a horizontal line, mark the appropriate level of water that must be filled inside the steam generator.

Any horizontal line between *AB* and *CD* .....(02)

(ii) A thermometer has to be inserted into the steam generator. Using a small cross (×) mark the appropriate position of the bulb of the thermometer that must be placed inside the steam generator.

Any indication of a cross above the drawn water level .....(02)

(Correct position of the bulb of the thermometer is accepted; No marks for a cross drawn below the water level)

(iii) The accurately measured temperature of steam in this experiment is 99·0 °C not 100·0 °C. What can be the reason for this?

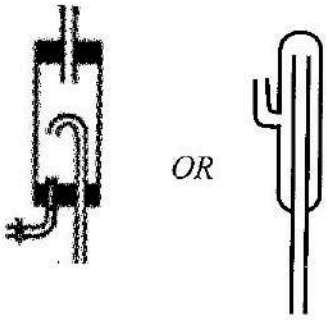
(The temperature of steam/boiling point of water) depends on atmospheric pressure/altitude/height (from sea level) *OR* the location of the experiment *OR* the location of the school

.....(02)

(*b*) (i) Name the item that you use to avoid mixing condensed steam into water in the calorimeter.

A steam trap .....(02)

(ii) Draw the item mentioned in (*b*) (i) above in the appropriate place in figure (1) with the correct connection.



... .(03)

[01 mark for the correct diagram; 01 mark for the correct connection to the rubber tube; 01 mark for lower end of the tube above the water level in the calorimeter]

(Overflow tube is not essential)

(*c*) Two thermometers *A* and *B* are available for the experiment.

The range of thermometer *A*: –10 °C to 110 °C

The range of thermometer *B*: –10 °C to 60 °C

Which thermometer must be used to measure temperature of water in the calorimeter?

B OR temperature range (from $-10\ ^0$C to) 60 ^{0}C .....(01)

(*d*) What are the mass measurements that you would take in this experiment? Give them in order of measurements.

(1) The mass of the empty calorimeter and stirrer/calorimeter with contents

(2) The mass of the calorimeter, stirrer and water

(3) The total/final mass of the system/mixture (after adding steam) .....(03)

[03 marks for all three correct answers in correct order; 02 mark for all three correct answers but wrong order ; 01 mark for two answers in correct order]

(*e*) What are the experimental steps that you would take to measure the last temperature reading of water in this experiment?

(1) Stop passing steam into the water. .....(01)

(2) Stir well and record the highest/maximum temperature of the mixture.

.....(01)

(*f*) The room temperature and the initial temperature of water are θ and θ_1 respectively. Write down an expression for the value of last temperature measurement θ_2 of water in terms of θ_1 and θ to minimize heat exchange with the surroundings.

$$\theta - \theta_1 = \theta_2 - \theta$$

$$\theta_2 = 2\theta - \theta_1 \quad \text{.....(01)}$$

(*g*) (i) Is it possible to use a glass beaker instead of a copper calorimeter in this experiment?
Possible/Not possible (Underline the correct answer.) .....(01)

(ii) Give the reason for the above answer.

The specific heat capacity of copper is very low/ The specific heat capacity of glass is higher compared to copper *OR* the absorption and liberation of heat will be more from a glass beaker compared to a copper calorimeter during the experiment *OR* the temperature will not be uniform on the walls of a glass beaker *OR* there will be a temperature gradient across the wall *OR* temperature of water will not be equal to the temperature of the glass beaker

.....(01)

(*h*) Disregard this part.

3. You are required to determine the refractive index of material of a glass prism using a laboratory spectrometer.

(*a*) Draw the path of a monochromatic ray incident on the surface *AC* undergoing minimum deviation through the prism shown in figure (1). Also mark the angle of incidence (*i*) and the angle of refraction (*r*) of the ray at the surface *AC*.

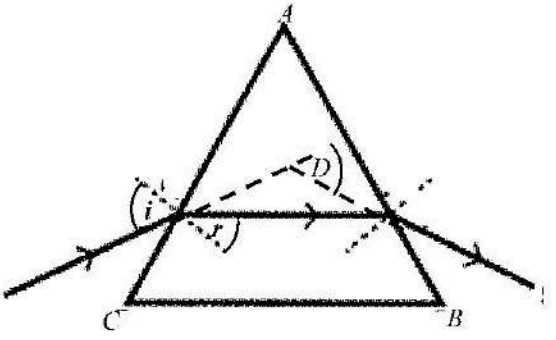


Figure (1)

Path of a ray passing through the prism symmetrically (parallel to the surface CB) with at least drawing an arrow head. .....(01)

Marking i and r on surface AC. .....(01)

(*b*) **Mark the angle of minimum deviation (D) of the ray in the above figure (1).**

Marking the angle D .....(01)

(*c*) **Write down an expression for the refractive index (n) of material of prism in terms of angle of prism A and D.**

$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$ .....(02)

(*d*) **Give the experimental steps needed for adjusting the telescope of the spectrometer.**

Move the eyepiece (back or forth) until a clear/sharp image of the cross-wires are clearly seen/observed through it .....(01)

Direct the telescope on to a distant object and rotate the adjusting knob of the telescope until a clear image (with sharp edges) is seen. .....(01)

(*e*) **A student argues that a beam of light from a bright filament bulb can be used for levelling the prism table. Do you agree?**

Yes/agree .....(01)

Give the reason.

Since reflection of light is used to level the prism table a bright filament bulb can be used. .....(01)

(*f*) **After adjusting all the parts of the spectrometer how do you experimentally obtain the minimum deviation position for a monochromatic ray of light?**

Use a sodium lamp (*OR* mercury lamp) .....(01)

Keep the prism in the middle of the prism table so that the angle of incidence is small (about 10°). .....(01)

Rotate the prism table in the increasing direction of the angle of incidence while looking through the telescope. .....(01)

The position where the image of the slit (*OR* face of the collimator) returns/turns back is the minimum deviation position. .....(01)

(*g*) When the telescope is fixed to the minimum deviation positon, the circular scale and the vernier scale positions are shown in figure (2).

What is the reading of this position?

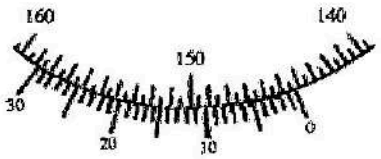


Figure (2)

144°15' .....(02)

(*h*) After removing the prism from the prism table, the direct reading of the telescope is measured as 104°55′. Find the value of *D*. The 360° mark of the circular scale has not crossed when taking measurements.

$D = 144°15' - 104°55'$ (for the subtraction) .....(01)

$= 39°20'$ .....(01)

(*i*) If the prism angle is $A = 60°00'$, calculate the refractive index (n) of material of prism. (Use the natural sine table for your calculation.)

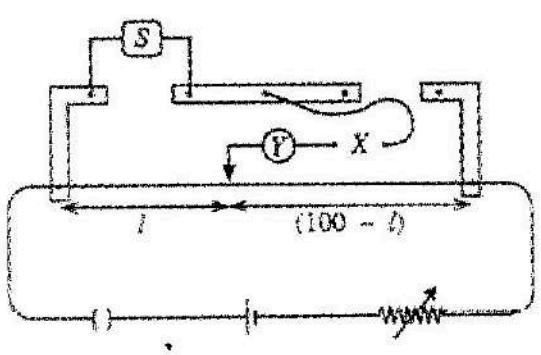
$(D + A)/2 = (39°20' + 60°00')/2$ (for the addition and division) .....(01)

$= 49°40'$

$$n = \frac{\sin 49°40'}{\sin 30°}$$

$= 1.52$ (1.51 – 1.53) .....(02)

4. Figure (1) shows an experimental setup that can be used to determine the temperature coefficient of resistance (α) of material of a thin wire using a metre bridge. An electrically insulated uniform wire of length 5·0 m and diameter 0·1 mm, is wound around a cylindrical piece of plastic rod to make a coil. The resistivity of the material of the wire is $1{\cdot}5 \times 10^{-8}\ \Omega$ m at 30 °C. A suitable resistance S is connected across the left gap of the bridge.



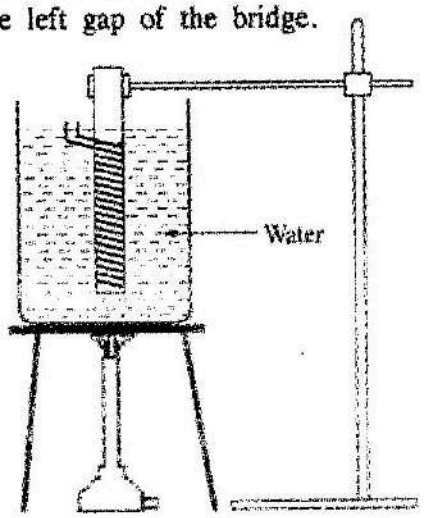


Figure (1)

(*a*) Find the resistance of the coil of wire at 30 °C. (Take $\pi = 3$.)

$R = \rho \frac{l}{A}$ .....(01)

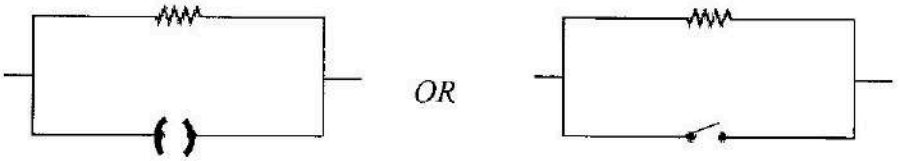
$= 1.5 \times 10^{-8} \frac{5}{\pi \left(\frac{0.0001}{2}\right)^2}$

$= 10.0\ \Omega$ .....(01)

(*b*) What is the measuring instrument labelled as '*Y*' in figure (1)?

Center zero galvanometer .....(01)

(*c*) (i) In the space given below, draw a diagram of the circuit that should be connected across the gap '*X*' in figure (1).



.....(01)

(ii) What is the purpose of the circuit that you have drawn in (*c*) (i) above?

To protect the galvanometer (from high currents) *OR* to avoid passing high currents through the galvanometer *OR* to prevent burning of the galvanometer.

.....(01)

(*d*) Copper wires have to be used to connect the coil of wire to the metre bridge. What type of wires are suitable for this?

Short length .....(01)

Large cross sectional area/thick wires .....(01)

(*e*) What are the other essential instrument and item needed in this experiment?

Instrument : Thermometer .....(01)

Item : Stirrer .....(01)

(*f*) (i) If the resistance of the coil at a given temperature θ (°C) is R_θ and the corresponding balance length of the metre bridge wire is l (cm), write down an expression for $\frac{R_\theta}{S}$ in terms of l. Neglect end corrections of the metre bridge wire.

$\frac{R_\theta}{S} = \frac{100-l}{l}$ .....(01)

(ii) Write down an expression for the resistance R_θ in terms of α, the resistance R_0 at $\theta = 0\,°C$ and θ.

$R_\theta = R_0\,(1 + \alpha\,\theta)$ .....(01)

(iii) By combining expressions written in (*f*) (i) and (ii) above, obtain an expression needed to draw the straight line graph of $\left(\frac{100}{l} - 1\right)$ versus θ.

$\frac{100}{l} - 1 = \frac{R_0(1+\alpha\theta)}{S}\ OR\ \frac{100}{l} - 1 = \frac{R_0\alpha}{S}\theta + \frac{R_0}{S}$ .....(01)

(iv) Using the parameters of the expression written in (*f*) (iii) above, write down expressions for the gradient (m) and intercept (c) of the graph.

$m = \alpha\frac{R_0}{S}$ .....(01)

$c = \frac{R_0}{S}$ .....(01)

(v) Write down an expression for α in terms of m and c.

$\alpha = \frac{m}{C}$ .....(01)

(g) Calculate α using the following graph shown in figure (2).

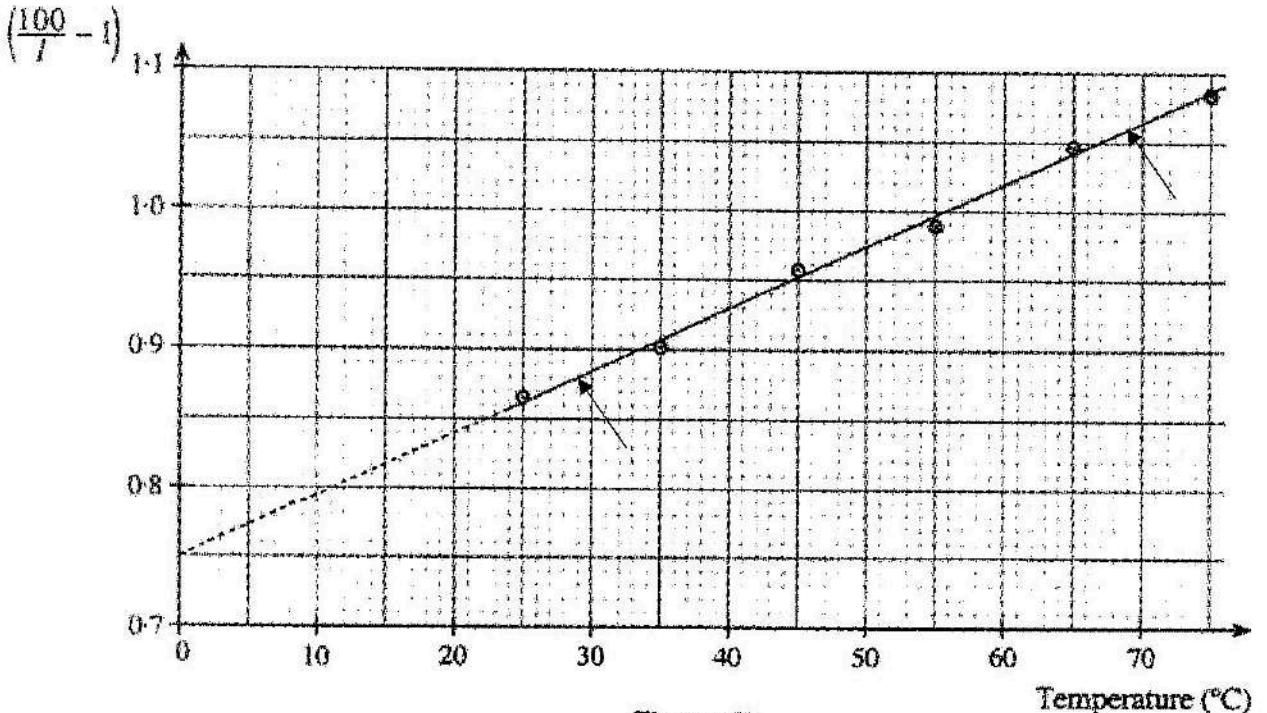


Figure (2)

Selecting the lower point as (29, 0.88) .....(01)

Selecting the higher point as (69, 1.06) .....(01)

(No marks for any other points)

Gradient $= \frac{1.06-0.88}{(69-29)}$ (for gradient calculation) .....(01)

$= \frac{0.18}{40}\ {}^{0}C^{-1}$

Intercept of the graph = 0.75

$$\alpha = \frac{0.18/_{40}}{0.75}$$

$\alpha = 6.0 \times 10^{-3}\ {}^{0}C^{-1}$ $(0.006\ {}^{0}C^{-1})$ .....(02)

(Deduct 01 mark for incorrect unit; K unit is not correct)

{If a student has obtained the correct answer for α selecting different points on the line for gradient calculation, award 03 marks only. i.e. for gradient calculation and the final answer}

Answer four questions only.
($g = 10\,m\,s^{-2}$)

- **Note:** For an example the number 65210 can be written as $6{\cdot}52 \times 10^4$ in scientific notation after rounding off to two decimal places.

5. Read the following passage and answer the questions.

In ice skating, the blade of an ice skate shown in figure (1) exerts pressure on the ice and melts a thin layer of ice providing lubrication between the ice and the blade. This is called 'pressure melting'. The bottom surface of the skate blade is 30 cm long and 1 mm wide. A human who exerts his weight on one ice skate can create pressure of up to 20 times the normal atmospheric pressure. The coefficient of friction between ice and the blade is almost zero. Therefore the only way to move forward is by pushing the ice wall not melted with a force by the skate blade as shown in figure (2).



Figure (1)

As the ice skater pushes with his right leg from behind, a force F is exerted on the skate blade by the ice. The component of the force F in the direction of motion pushes the skater forward. At the same time, his left leg with skate is either raised or gliding on the ice surface. As the skater moves forward he changes the above action to the left leg and pushes the ice with that one while raising the right leg. The process is continuously repeated.

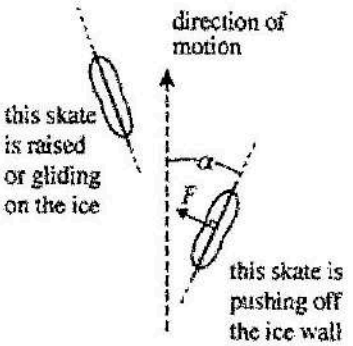


Figure (2)

Figure (3) shows the forces acting on a skater of mass m as he moves in a circular path on a flat horizontal ice surface with a constant speed. Here, G is the center of mass of the skater, P is the point of contact between a skate and ice surface, and L is the distance between P and G. The horizontal and vertical components of the force exerted on skate from ice are F_X, and F_Y respectively. The radius of the circular path is R.

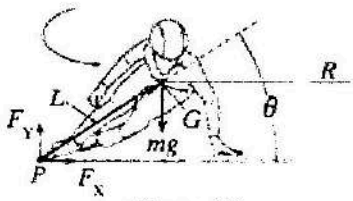


Figure (3)

In order to achieve a spin motion of a skater, a special blade is used which has small, jagged spikes at the front end as shown in figure (4). Spins are made by this jagged spikes digging into the ice and creating necessary torque.

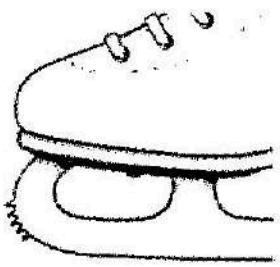


Figure (4)

(*a*) What is meant by 'pressure melting'?

(*b*) (i) If a person of mass 60 kg wearing regular shoes stands on ice surface with one foot, what is the pressure exerted by him on the ice surface? The surface area of the bottom surface of one shoe is 300 cm^2.

(ii) If he wears a skate instead of the regular shoe, what is the pressure exerted by him on the ice surface? Take the dimensions of the skate blade from the passage. Assume that the bottom surface of the skate blade is rectangular.

(iii) Hence, show that the pressure obtained in (*b*) (ii) above is 20 times the atmospheric pressure. (Atmospheric pressure is $1{\cdot}0 \times 10^5$ Pa.)

(*c*) How does a skater move forward on an ice surface?

(*d*) (i) What is the component of the force that points in the direction of motion of the skater? Write down your answer in terms of F and α.

(ii) Is it possible for him to move forward if the angle α is zero? Give the reason for your answer.

(*e*) (i) If the average force continuously applied by changing legs is 180 N, determine the acceleration (a) of the skater of mass 60 kg along the direction of motion. Take $\alpha = 30°$. Assume that there are no other resistive forces acting on him.

(ii) If he starts from rest what is his speed (v) after accelerating for 5 s?

(*f*) Using figure (3), show that the speed v' of the skater moving in a circular path is given by $v' = \sqrt{\frac{gR}{\tan\theta}}$.

(*g*) What is the purpose of having jagged spikes on the blade as shown in figure (4)?

(*h*) A female figure skater of mass 60 kg with her arms extended horizontally as shown in figure (5), spins about a vertical axis with an angular speed of 60 rpm. Then she fully retracts her arms, by bringing arms very close to her body as shown in figure (6). Extended arms can be considered as uniform rods each of length 60 cm and mass 7 kg. Remainder of the body without arms can be considered as a solid cylinder of mass 46 kg and radius 20 cm. The body with fully retracted arms can be considered as a solid cylinder of mass 60 kg and radius 20 cm. The moment of inertia of a rod of mass M and length L about an axis perpendicular to the length at one end is given by $\frac{1}{3}ML^2$. The moment of inertia of a solid cylinder of mass M and radius R about the central axis is given by $\frac{1}{2}MR^2$. (Take $\pi = 3$)

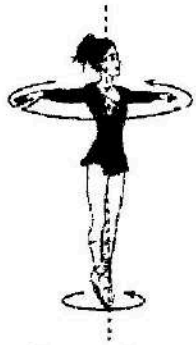


Figure (5)



Figure (6)

(i) Determine the total moment of inertia of the female skater about the rotating axis when her arms are fully extended. Neglect the distance between rotating axis and shoulder joint.

(ii) Determine the moment of inertia of her about the rotating axis when her arms are fully retracted.

(iii) Hence, calculate the angular speed of her in rpm when she fully retracts her arms.

(iv) Name the conservation law that you have used to find out the answer to (*h*)(iii) above.

(v) Calculate the initial and final rotational kinetic energies of her. How do you explain the change in the rotational kinetic energy?

(vi) Starting from rest if she takes 10 s to achieve her angular speed of 60 rpm what is the torque that has to be applied on the jagged spikes from ice? Assume that the angular acceleration of her is constant throughout the process.

(a) Exerting pressure on an ice surface and melting a thin layer of ice .....**(01)**

(b) (i) Pressure exerted $= \frac{60\times10}{300\times10^{-4}}$

$= 2 \times 10^4$ N m^{-2} .....**(01)**

(ii) Pressure exerted with the skate $= \frac{60\times10}{30\times10^{-2}\times10^{-3}}$ .....**(01)**

(for the correct substitution)

$= 2 \times 10^6$ N m^{-2} .....**(01)**

(iii) This (2×10^6) is 20 times more than the atmospheric pressure. .....**(01)**

(*c*) By pushing the ice wall not melted by the skate blade. .....(01)

(*d*) (i) $F_1 = F\sin\alpha$ .....(01)

(ii) Not possible .....(01)

Then the force/component of the force along the direction of motion will be zero.

.....(01)

(*e*) (i) Applying $F = ma$ to the skater

$180 \times \sin 30 = 60 \times a$ .....(01)

(for the correct substitution)

$a = 1.5\ \text{m s}^{-2}$ .....(01)

(ii) Applying $v = u + at$

$v = 1.5 \times 5$ .....(01)

(for the substitution)

$= 7.5\ \text{m s}^{-1}$ .....(01)

(*f*) Applying $\rightarrow F = ma$

$F_x = m\frac{v^2}{R}$ .....(01)

Similarly $F_y = mg$ .....(01)

$\therefore \frac{F_y}{F_x} = \frac{gR}{v^2}$

Taking moments about G, $F_x \times L\sin\theta = F_y \times L\cos\theta$ .....(01)

$\therefore \frac{F_y}{F_x} = \tan\theta$

$v = \sqrt{\frac{gR}{\tan\theta}}$

(*g*) In order to achieve a spin motion. .....(01)

(*h*) (i) Moment of inertia of two arms $= 2 \times \frac{1}{3} \times 7 \times 0.6^2$ .....(01)

(for the correct substitution)

$= 1.68 \text{ kg m}^2$

Moment of inertia of the body $= \frac{1}{2} \times 46 \times 0.2^2$ .....(01)

(for the correct substitution)

$= 0.92 \text{ kg m}^2$

Total moment of inertia of the skater $= 1.68 + 0.92$ .....(01)

(for addition)

$= 2.6 \text{ kg m}^2$ .....(01)

(ii) Moment of inertia of the skater about when her arms are fully retracted $= \frac{1}{2} \times 60 \times 0.2^2$

$= 1.2 \text{ kg m}^2$ .....(01)

(iii) Angular speed in rpm when the skater fully retracts her arms $= \frac{2.6 \times 60}{1.2}$ .....(01)

(for substitution)

$= 130$ rpm .....(01)

(iv) Conservation of angular momentum .....(01)

(v) 60 rpm = 1 revolution per second

Initial rotational speed in rad $s^{-1} = 2 \times 3 \times 1 = 6$ rad s^{-1}

Initial kinetic energy of the skater $= \frac{1}{2} \times 2.6 \times 6^2$

$= 46.8 \text{ J}$ .....(01)

Final kinetic energy of the skater $= \frac{1}{2} \times 1.2 \times 13^2$

$= 101.4 \text{ J}$ .....(01)

Work done by the skater in pulling her arms inwards. .....(01)

(vi) Applying $\omega = \omega_0 + \alpha t$

$6 = \alpha \times 10$

$\alpha = 0.6 \text{ rad s}^{-2}$ .....(01)

Applying $\tau = I\alpha$

$\tau = 2.6 \times 0.6$

$\tau = 1.56 \text{ N m}$ .....(01)

6. (a) If the intensity at a given point produced by a sound source is I and the threshold of hearing is I_0, define the sound intensity level (β) at the point by an equation.

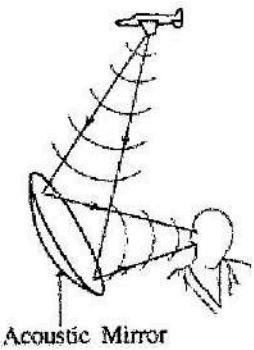
(*b*) Sound intensity produced by an airplane engine at a point is $2{\cdot}0 \times 10^{-2} \text{ W m}^{-2}$. $I_0 = 1{\cdot}0 \times 10^{-12} \text{ W m}^{-2}$ and $\log 2 = 0{\cdot}3$. You may use $\log(ab) = \log(a) + \log(b)$

(i) Find the sound intensity level at that point.

(ii) If the airplane has two engines, what is the total sound intensity level at the same point? Assume that the two engines are at equidistance from the point.

(*c*) (i) At the beginning of World War II, there were no radar facilities available, hence sound waves generated by airplanes were used to detect airplanes. If sound intensity level at a human ear must be at least 30 dB to detect an airplane, find the corresponding minimum sound intensity that should be generated at the ear by the airplane.

(ii) Acoustic mirrors were used to reflect and focus sound waves to increase the sensitivity of detection. An acoustic mirror with effective surface area of 4 m^2 concentrates sound into an ear with effective surface area of 10 cm^2 as shown in the figure. What should be the minimum incident sound intensity at the acoustic mirror to detect an airplane? Neglect the sound energy absorption by the mirror. Assume that there is no loss of sound energy during the propagation between the acoustic mirror and the ear.



(iii) An airplane generates a sound power of 480 W from its engines. Assume a uniform spherical sound distribution. (Take $\pi = 3$.)

(I) If the atmosphere absorbs 95% of sound energy during the propagation between the airplane and the ear, find the maximum distance that an airplane can be detected without the acoustic mirror using the value obtained in (*c*) (i) above. (Take $\sqrt{5} = 2{\cdot}24$.)

(II) If the atmosphere absorbs 99.9% of sound energy during the propagation between the airplane and the acoustic mirror, find the maximum distance that an airplane can be detected with the acoustic mirror using the value obtained in (*c*) (ii) above. Assume that there is no loss of sound energy during the propagation between the acoustic mirror and the ear.

(*d*) An air surveillance person on the ground identifies an airplane, which is flying along a straight-line path above him parallel to ground at a vertical height of 3000 m with a velocity of 125 m s^{-1}. At time $t = 0$, the horizontal distance from the airplane to the observer is 4000 m. The frequency of sound generated by the airplane is 100 Hz. Assume the speed of sound in air is 300 m s^{-1}.

(i) Find the frequency of sound heard by the surveillance person on ground at times $t = 0$ s, $t = 32$ s and $t = 64$ s.

(ii) Sketch the variation of observed frequency (f) against time (t) for the above situations.

(*e*) A supersonic jet flies in a straight line path at velocity u parallel to the ground at a height of 3000 m. The speed of sound in air at that height is v.

(i) Draw circular wave-fronts emitted and then transmitted from the jet for situations $u < v$, $u = v$ and $u > v$.

(ii) For the situation $u > v$, the Mach number M and Mach angle α (half angle at the vertex of the Mach cone) can be defined for a jet as $M = \frac{u}{v}$ and $\sin\alpha = \frac{v}{u}$ respectively. If the velocity of the jet is Mach 2, how long after the plane has passed directly overhead will an observer hear the sonic boom? Speed of sound v at that height is 300 m s^{-1}. Take $\sqrt{3} = 1.73$.

(*a*) $\beta = 10\log\left(\frac{I}{I_0}\right)$ .....(02)

(*b*) (i) $\beta = 10\log\left(\frac{2\times10^{-2}}{1\times10^{-12}}\right)$ .....(01)

$= 10[\log(2) + \log 10^{10}] = 10\,[\log(2) + 10] = 10 \times 10.3$

$= 103$ dB .....(01)

(ii) $\beta = 10\log\left(\frac{2\times2\times10^{-2}}{1\times10^{-12}}\right)$ .....(01)

$= 10\,[\log(2) + \log(2) + 10] = 10 \times 10.6$

$= 106$ dB .....(01)

(*c*) (i) If I is the minimum sound intensity

$30 = 10\log\left(\frac{I}{1\times10^{-12}}\right)$ .....(01)

$I = 10^{-9}$ W m^{-2} .....(01)

(ii) If I' is the minimum sound intensity at the mirror

$I' \times 4 = 10^{-9} \times 10 \times 10^{-4}$ .....(01)

$I' = 2.5 \times 10^{-13}$ W m^{-2} .....(01)

(iii) (I) If d is the maximum distance without the mirror

$\frac{480}{4\pi(d)^2} \times 0.05 = 10^{-9}$ .....(02)

(01 mark for $\frac{480}{4\pi(d)^2}$; 01 mark for the rest)

$d^2 = 40 \times 0.05 \times 10^9$

$= 4 \times 5 \times 10^8$

$d = 2\sqrt{5} \times 10^4 = 2 \times 2.24 \times 10^4$

$d = 4.48 \times 10^4$ m (44.8 km) .....(01)

(II) If d' is the maximum distance with the mirror

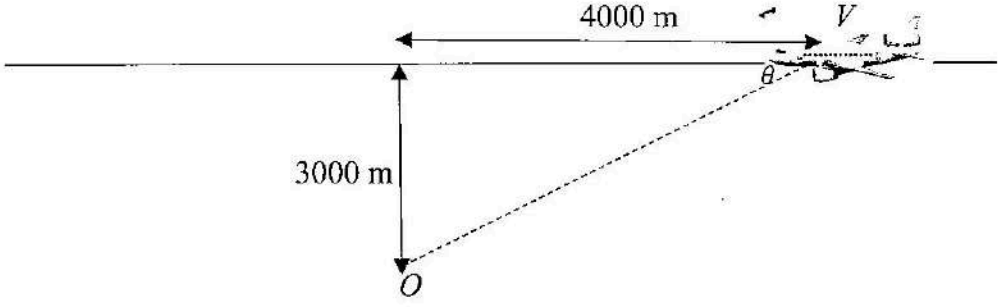
$\frac{480}{4\pi(d')^2} \times 0.001 = 2.5 \times 10^{-13}$ .....(01)

$d'^2 = \frac{4}{25} \times 10^{12}$

$d' = \frac{2}{5} \times 10^6$

$d' = 4 \times 10^5$ m (400 km) .....(01)

(*d*)



(i) If V' is the velocity component of the plane towards the observer at $t = 0$

$V' = V \cos\theta$ .....(01)

$V' = 125 \times \frac{4}{5}$ .....(01)

$f' = \frac{v}{v - v_s} f$

$= \frac{300}{300-100} \times 100$ .....(01)

$f' = 150$ Hz .....(01)

At $t = 32$ s, distance travelled by the plane $= 125 \times 32 = 4000$

Therefore the plane is right above the observer,

$V' = 0$ (for identifying $V' = 0$) .....(01)

$\therefore f' = 100$ Hz .....(01)

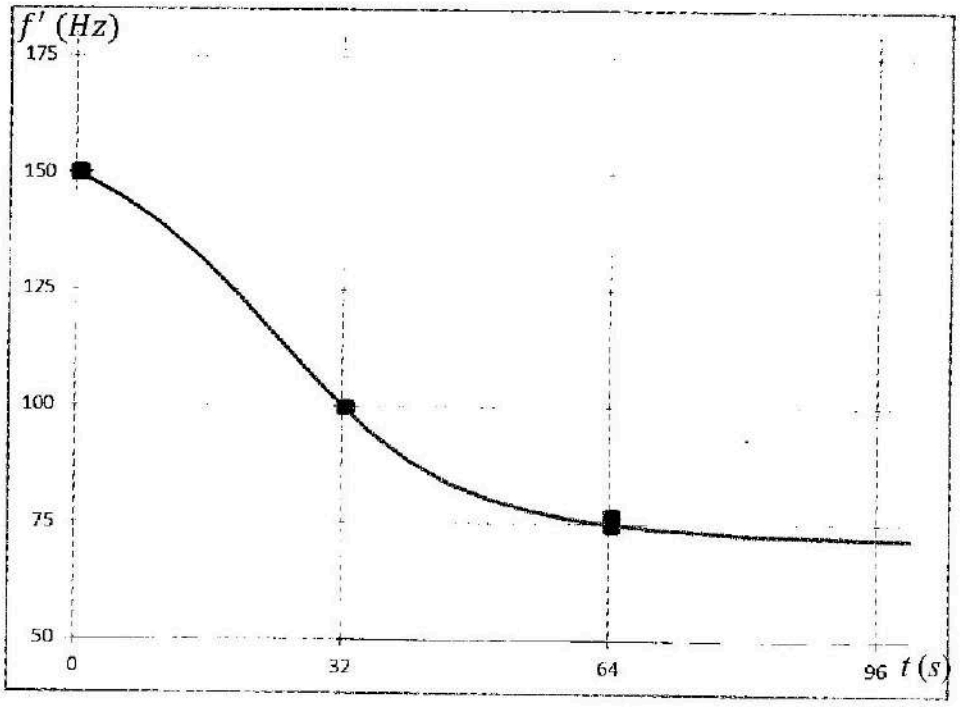
At $t = 64$ s, the plane is moving away from the observer

$$f' = \frac{v}{v+v_s} f$$

$$= \frac{300}{300+100} \times 100$$ .....(01)

$f' = 75$ Hz .....(01)

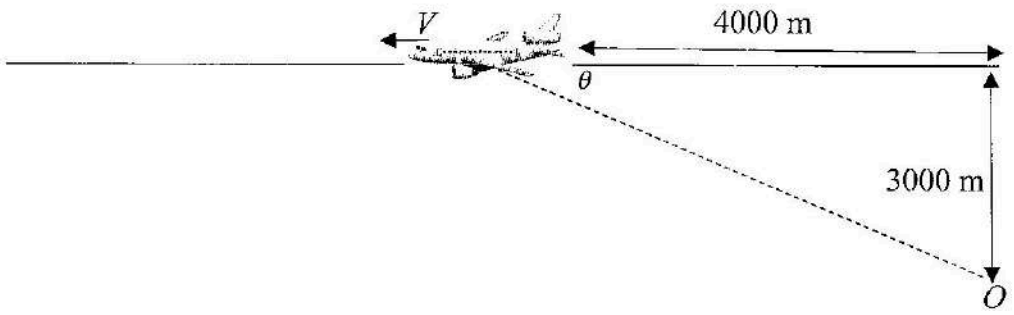
(ii)



.....(02)

(01 mark for the shape for approaching; 01 mark for the shape for receding; Labelling the axes and marking the coordinates are not essential)

Alternative answer:



(*d*) (i)If V' is the velocity component of the plane away from the observer at $t = 0$

$V' = V\cos\theta$ .....(01)

$V\cos\theta = 125 \times \frac{4}{5}$ .....(01)

$f' = \frac{v}{v+v_s} f$

$= \frac{300}{300+100} \times 100$ .....(01)

$f' = 75$ Hz .....(01)

At $t = 32$ s, distance travelled by the plane $= 125 \times 32 = 4000$

$V' = V\cos\theta$

$V\cos\theta = 125 \times \frac{8000}{\sqrt{3000^2+8000^2}}$ .....(01)

$= 117$ m s^{-1}

$f' = \frac{v}{v+v_s} f$

$= \frac{300}{300+125 \times \frac{8000}{\sqrt{3000^2+8000^2}}} \times 100$ .....(01)

$f' = 71.9$ Hz

At $t = 64$ s, distance travelled by the plane $= 125 \times 64 = 8000$

$$V' = V\cos\theta$$

$$V\cos\theta = 125 \times \frac{12000}{\sqrt{3000^2+12000^2}} \quad \text{.....(01)}$$

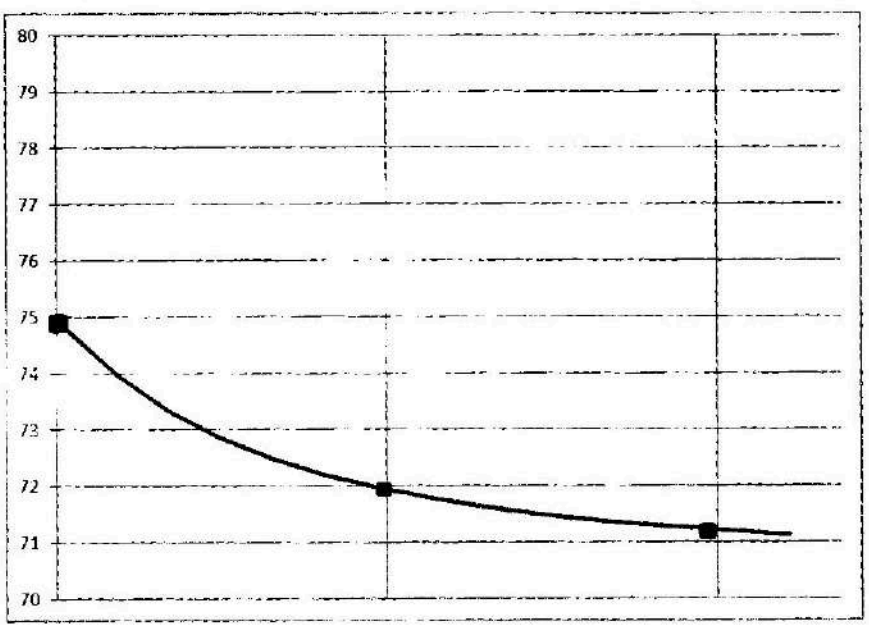
$$= 121 \text{ m s}^{-1}$$

$$f' = \frac{v}{v+v_s} f$$

$$= \frac{300}{300+125\times\frac{12000}{\sqrt{3000^2+12000^2}}} \times 100 \quad \text{.....(01)}$$

$$f' = 71.3 \text{ Hz}$$

(ii)

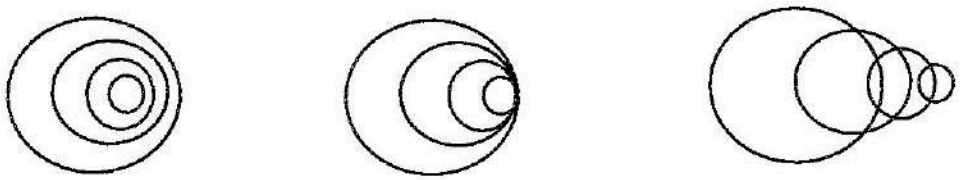


.....(02)

(02 or zero; Labelling the axes and marking the coordinates are not essential)

(*e*) (i)

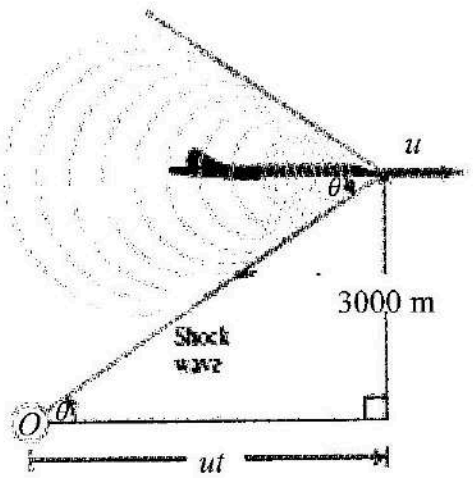
$u < v$ $u = v$ $u > v$



.....(03)

(01 mark for each correct shape; look for at least three circles per diagram)

(ii)



$u = 600 \text{ m s}^{-1}$

$\sin(\theta) = \frac{300}{600}$

$\theta = 30^0$

$\tan(\theta) = \frac{3000}{ut}$ *OR* $\tan(30^0) = \frac{3000}{600t}$ .....(01)

$t = \frac{3000}{600}\sqrt{3}$

$t = 8.65 \text{ s}$ .....(01)

7. (*a*) Define the coefficient of surface tension.

(*b*) Three long glass capillary tubes are vertically dipped in different liquids having the angle of contact (i) 0°, (ii) 90° and (iii) 135° so that a half of the tube is inside the liquid. Draw a sketch showing the shape of the liquid meniscus inside the tube, the height of the liquid column and the shape of the liquid surface outside near the tube for each case.

(*c*) Derive an expression for the maximum radius (r_m) of a tiny solid sphere that can be floated without piercing the surface of a liquid with coefficient of surface tension T. The density of the material of the sphere is β and it is greater than the density of the liquid. Assume that the contact angle between the material of the sphere and liquid is zero. The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

(*d*) Hay's test is done to detect the presence of bile salts in urine to identify patients having jaundice. Bile salts reduce the surface tension of urine. Sulphur powder having uniform spherical particles is sprinkled on a sample of urine taken for the Hay's test.

(i) Using the expression derived in (*c*) above, calculate the maximum radius (r_m) of spherical sulphur particles that can be floated on normal urine. Density of sulphur is 2000 kg m^{-3} and the surface tension of normal urine is $6{\cdot}5 \times 10^{-3}$ N m^{-1}. Give your answer to one decimal place in mm.

(ii) Sulphur particles will sink if bile salts are present and the person is identified as positive for jaundice. For Hay's test according to the value calculated in (*d*) (i) above sulphur particles of radius $0{\cdot}9\,r_m$ is used. If these particles just sink in the urine sample of a patient having jaundice, calculate the surface tension of affected urine. Round off your answer to one decimal place in scientific notation.

(*e*) If a capillary tube of radius 0·4 mm is vertically dipped in unaffected urine sample calculate the capillary rise. Density of normal urine is 1020 kg m^{-3}. The angle of contact between urine and glass is 30°. Give your answer to nearest integer in mm. (Take $\sqrt{3}$ = 1·73)

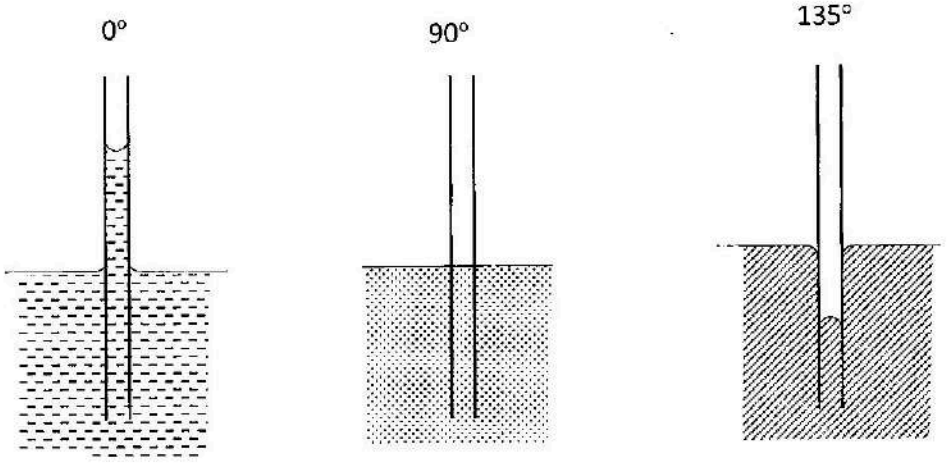
(*f*) Another test method can be designed using an electrical liquid sprayer gun producing tiny urine droplets with identical radii in one second. What will be the ratio of power needed for forming droplets from urine with bile salt sample to the normal urine sample? Assume that the densities of urine in both samples are the same. Give your answer to two decimal places.

(*a*) The coefficient of surface tension of a liquid is the force acting on the surface of the liquid at right angles to one side of an imaginary straight line of unit length in the surface of the liquid.

.....**(02)**

(02 or zero)

(*b*)

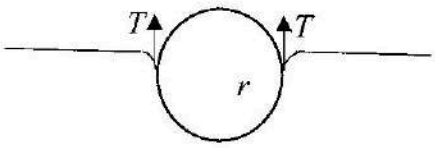


.....**(06)**

Look for meniscus shape, height of the liquid column and shape of the liquid surface outside the tube. If all are correct 02 marks for each. Deduct 01 mark for each mistake.

(*c*) The sphere is not submerged and therefore, up thrust does not act on the sphere.

The weight is balanced by the surface tension forces



OR

$\frac{4}{3}\pi r_m^3 \beta g = 2\pi r_m T$ .....**(03)**

(01 mark for L.H.S. term; 01 mark for R.H.S. term; 01 mark for equating)

$$r_m = \sqrt{\frac{3T}{2\beta g}}$$ .....**(02)**

(*d*) (i) $r_m = \sqrt{\frac{3\times6.5\times10^{-3}}{2\times2000\times10}}$ .....**(02)**

(for correct substitution)

$= \sqrt{48.75} \times 10^{-4} \approx \sqrt{49} \times 10^{-4}$

$r_m = 0.7$ mm .....**(02)**

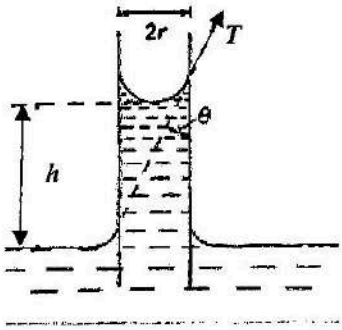
(ii) $T = \frac{r^2 2\beta g}{3}$

$= \frac{(0.9\times0.7\times10^{-3})^2\times2\times2000\times10}{3}$ .....**(02)**

(for substitution)

$= 5.3 \times 10^{-3}$ N m^{-1} (*OR* 5.26×10^{-3} N m^{-1}) .....**(02)**

(e)



$$h = \frac{2T\cos\theta}{r\rho g} \quad \text{.....(02)}$$

$$= \frac{2\times6.5\times10^{-3}\times\sqrt{3}}{2\times0.4\times10^{-3}\times1020\times10} \quad \text{.....(01)}$$

(for substitution)

$= 3$ mm OR $(2.7 - 2.8)$ mm .....(02)

(f) Energy needed to make a drop of radius $r = 4\pi r^2 T$

Since energy per second is power (p) and the radii are the same,

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

Required $\frac{P_1}{P_2}$ ratio $= \frac{5.3\ (or\ 5.26)}{6.5}$ .....(02)

(for substitution)

$= 0.82\ (0.81 - 0.82)$ .....(02)

8. (a) Consider a metal conductor in the form of a rectangular thin slab having width w and thickness t as shown in figure (1). A constant current I flows in $+x$ direction and a uniform magnetic field of flux density B acts perpendicular to the plane of the slab, in $+y$ direction. The drift velocity of electrons is v_d. After the steady state is reached negative charges accumulate at the upper surface of the slab, leaving positive charges at the lower surface. Then a potential difference will be established between upper and lower surfaces of the slab which is called the Hall voltage V_H.

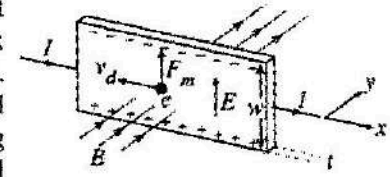


Figure (1)

(i) Derive an expression for the Hall voltage V_H in terms of the magnetic flux density B, the current I, the number of moving electrons per unit volume n in the conductor, electron charge e and thickness t of the slab.

(ii) If $B = 0{\cdot}4$ T, $I = 32$ A, $n = 10^{28}$ m^{-3}, $e = 1{\cdot}6 \times 10^{-19}$ C and $t = 2$ mm, determine V_H.

(iii) Without changing anything else, if the entire conductor is moved in the $-x$ direction with a constant velocity equal to the drift velocity of electrons, what will happen to the magnitude of the Hall voltage? Give reasons for your answer.

(iv) When the slab is stationary as shown in figure (1), F_m and E denote the magnetic force acting on the electron and Hall electric field intensity respectively. If the charge carriers are positively charged instead of negatively charged, what will happen to the directions of each v_d, F_m and E? (will change or will not change)

(*b*) Hall-effect sensors operate by detecting voltage changes when placed in a magnetic field. In order to generate a uniform magnetic field, two identical circular coils, each having the same radius, same number of turns, and identical currents, placed at a distance equal to the radius as shown in figure (2) can be used. The resulting magnetic flux density between the two coils is $1{\cdot}4B_0$, where B_0 is the magnetic flux density at the center of a single coil.

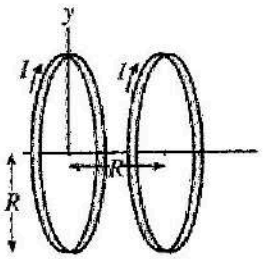


Figure (2)

(i) Starting from Biot-Savart law, obtain an expression for the magnetic flux density (B_0) at the center of a circular coil with number of turns N, radius R, and carrying current I. Name the other symbol in the expression.

(ii) If $N = 1000$, $I = 2$ A and $R = 0{\cdot}12$ m, calculate the magnetic flux density B_0 at the center of one coil. (Take $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1} and $\pi = 3$.)

(iii) Referring to the paragraph given in (*b*) above, calculate the value of the uniform magnetic flux density between the two coils if they are placed at a distance of 0·12 m.

(*c*) Hall-effect sensors are used in various applications to detect rotational speeds of rotating objects. Figure (3) shows a rotating wheel with four identical coils carrying identical currents mounted at equal intervals around the perimeter of the wheel. An additional coil (S), identical to those on the wheel carrying the same current is placed stationary with a Hall sensor next to it. When one of the coils on the rotating wheel aligns with the stationary coil S and the Hall sensor, a uniform magnetic field is established, allowing the Hall sensor to generate a voltage pulse. As the wheel rotates, each alignment produces a voltage pulse, enabling the detection of the rotational speed.

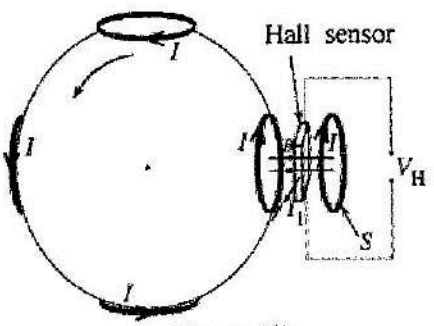


Figure (3)

(i) If the pulse frequency generated by the Hall sensor is f_0, write down an expression for the rotational frequency f of the wheel in terms of f_0.

(ii) If $f_0 = 240$ pulses per second, calculate the rotational speed ω of the wheel in rpm.

(iii) When the rotational speed of the wheel exceeds 7200 rpm an alarm has to be triggered. Determine the pulse frequency of the Hall sensor at which the alarm will be triggered.

(iv) In practice, semiconductors are used instead of metals to obtain large Hall voltages. Why does a semiconductor produce a large Hall voltage?

(a) (i) $eE = ev_dB$ (1) .....(01)

$I = newtv_d$ .....(01)

Substituting for v_d in equation (1)

$$E = \frac{I}{newt}B$$

But, $Ew = V_H$ .....(01)

$\therefore V_H = \frac{IB}{net}$ .....(02)

(ii) $V_H = \frac{32\times0.4}{10^{28}\times1.6\times10^{-19}\times2\times10^{-3}}$ .....(01)

(correct substitution)

$= 4\ \mu V\ (4 \times 10^{-6}\ V)$ .....(02)

(iii) The Hall voltage disappears/ becomes zero/vanishes .....(01)

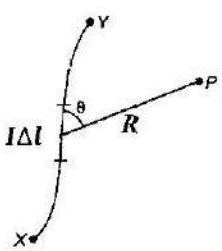
Because the electrons will be at rest with respect to the magnetic field *OR* the velocity of electrons will be zero/stationary with respect to the magnetic field. .....(01)

(iv) v_d : changes .....(01)

F_m : does not change .....(01)

E : changes .....(01)

(b) (i)



Biot-Savart law $\Delta B = \frac{\mu_0 I \Delta l \sin\theta}{4\pi R^2}$ .....(01)

(Accept μ also)

For a circular coil at the center $\theta = 90°$ .....(01)

and the circumference of the coil $= 2\pi R$ .....(01)

$\therefore B_0 = \frac{\mu_0 I 2\pi R}{4\pi R^2}$

$$B_0 = \frac{\mu_0 I}{2\pi R}$$

For a coil with N number of turns

$B_0 = \frac{\mu_0 NI}{2R}$ .....(02)

μ_o = permeability (of free space/air) .....(01)

(ii) $B_o = \frac{4\pi\times10^{-7}\times1000\times 2}{2\times0.12}$ .....(01)

(correct substitution)

$= 0.01$ T .....(02)

(iii) 0.014 T .....(01)

(c) (i) $f = \frac{f_0}{4}$ .....(02)

(ii) $\omega = \frac{240}{4} \times 60$ .....(01)

[Disregard the number 4 (accept any value) when awarding this mark]

$= 3600$ rpm .....(01)

(iii) $f_o = \frac{4\times7200}{60}$ .....(01)

[Disregard the number 4 (accept any value) when awarding this mark]

$= 480$ pulses per second .....(01)

(iv) Because n/ charge carriers (electrons) per unit volume/carrier density is small /less (compared to metals) .....(01)

9. Answer either part (A) or part (B) only.

Part (A)

(a) A thin heating element made from a conductive material of resistivity ρ has a shape of a rectangular strip with length l, width w and thickness t, as shown in figure (1).

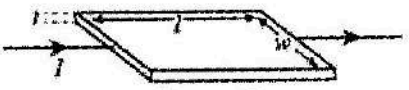


Figure (1)

(i) Write down an expression for the resistance R of the heating element in terms of ρ, l, w and t.

(ii) If $l = 100$ mm, $w = 20$ mm, $t = 5$ μm and $\rho = 8\times10^{-5}\ \Omega$ m calculate the resistance of the heating element.

(*b*) As shown in figure (2), a wearable heating pad is designed for localized heat therapy, using the above thin heating elements. The heating elements are arranged as shown in the figure, along with a 40 Ω standard resistor. The pad is connected to a 24 V d.c. source with negligible internal resistance. Thin heating elements are represented by rectangles. The heating pad should produce at least 7·0 W to deliver the required therapeutic heat.

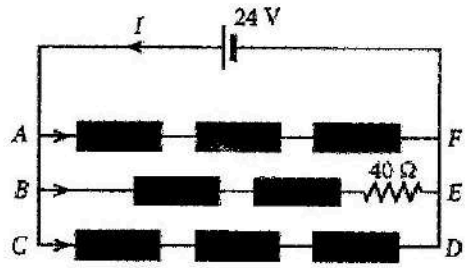


Figure (2)

(i) Calculate the resistance of branch *AF* and branch *BE* of the circuit.

(ii) Calculate the current through the branch *BE*.

(iii) Calculate the power dissipated in the branch *BE* and the entire circuit. Does the heating pad produce the required power?

(iv) Calculate the total power dissipation of the circuit if the thickness of all the heating elements is halved.

(v) If the length l is equal to the width w, show that the resistance is independent of the surface area (lw) of the heating element shown in figure (1).

(vi) Calculate the resistance per unit square of the top surface of the above heating element having thickness 5 μm.

(*c*) Assume that a heating pad contains resistive elements made of two thin layers deposited on top of each other.

Layer 1: Made of a material with a resistivity that remains constant with temperature.

Layer 2: Made of a material with same resistivity as Layer 1 initially but resistivity increases as temperature increases.

The heating pad operates using a constant voltage source. Giving reasons explain what will happen to the power dissipation of the modified heating pad over time.

(*d*) A d.c. source to supply power to circuits can be constructed using a suitable step down transformer. Here, a transformer is used to step down an input a.c. voltage of 240 V (r.m.s.) to an adjustable output a.c. voltage between 12 V (r.m.s.) and 48 V (r.m.s.). The transformer has 800 primary windings. At the output stage, the transformer output is converted to a d.c. voltage.

(i) Write down an expression for the ratio of primary voltage (V_p) to secondary voltage (V_S) of the transformer, in terms of the number of windings in the primary coil N_p and the secondary coil N_S.

(ii) If the r.m.s. voltage of the secondary coil can be varied between 12 V and 48 V, calculate the range of number of windings required in the secondary coil.

(iii) The d.c. output voltage is 80% of the r.m.s. output voltage of the transformer secondary coil. If the desired fully rectified d.c. output voltage is 24 V, calculate the output r.m.s. voltage of the transformer.

(iv) The transformer supplies power to a load that consumes 120 W at 24 V d.c. If the power loss due to Joule heating in the secondary is 10% of the power consumed by the load, calculate the r.m.s. output current of the transformer.

(*a*) (i) $R = \frac{\rho l}{wt}$ .....(01)

(ii) Resistance $= \frac{8\times10^{-5}\times100\times10^{-3}}{20\times10^{-3}\times5\times10^{-6}}$ .....(01)

(correct substitution)

$= 80\ \Omega$ .....(01)

(*b*) (i) $R_{AF} = 80 + 80 + 80$ .....(01)

(for the addition)

$= 240\ \Omega$ .....(01)

$R_{BE} = 80\ \Omega + 80\ \Omega + 40\ \Omega$ .....(01)

(for the addition)

$= 200\ \Omega$ .....(01)

(ii) For branch *BE*, $24 = I_{BE} \times 200$ .....(01)

(for substitution)

$I_{BE} = 120$ mA (0.12 A) .....(01)

(iii) $P_{BE} = I_{BE}^2 \times R_{BE}$ *OR* $\frac{V^2}{R_{BE}}$

$= (0.12)^2 \times 200$ *OR* $\frac{24^2}{200}$ .....(01)

(for substitution)

$= 2.88$ W .....(01)

$P_{AF} = \frac{24^2}{240}$ .....(01)

(for substitution)

$= 2.4$

$P = P_{AF} + P_{BE} + P_{CD} = 2.4 + 2.88 + 2.4$

$= 7.68$ W .....(01)

Yes (since $7.68 > 7$) .....(01)

(to earn this mark students should obtain 7.68 W as the answer)

(iv) Making the thickness halved causes the resistance of printed resistors to increase by 2 times. Hence power in AF and CD branches becomes half of the original value.

$R_{BE,new} = 80 \times 2 + 80 \times 2 + 40$ .....(01)
(for the addition)

$= 360\ \Omega$

$P_{BE,new} = 24^2/360$ *OR* $\left(\frac{24}{360}\right)^2 \times 360$ .....(01)

$= 1.6$ W

$P_{new} = \frac{2.4}{2} + \frac{2.4}{2} + 1.6$ .....(01)
(for the addition)

$= 4$ W .....(01)

(v) If $l = w$, $R = \frac{\rho l}{lt} = \frac{\rho}{t}$ .....(01)

This is independent of lw.

(vi) $R = \frac{8\times10^{-5}}{5\times10^{-6}}$

$= 16\ \Omega$ .....(01)

(*c*) Power dissipation in layer 1 does not change with time .....(01)

Power dissipation in layer 2 decreases with time .....(01)

Therefore power dissipation in the modified heating pad will decrease with time .....(01)

(*d*) (i) $\frac{V_p}{V_s} = \frac{N_p}{N_s}$ .....(01)

(ii) For 12 V, $N_s = N_p \times V_s / V_p = 800 \times 12 / 240$

$= 40$ turns .....(01)

For 48 V, $N_s = N_p \times V_s / V_p = 800 \times 48 / 240$

$= 160$ turns .....(01)

(iii) $V_{d.c.} = V_{s,r.m.s.} \times 0.8$

$V_{s,r.m.s.} = \frac{V_{d.c.}}{0.8} = \frac{24}{0.8}$ .....(01)

(for correct substitution)

$= 30$ V .....(01)

(iv) $I_{s\,r.m.s.} = P / V_{s,r.m.s.}$

$I_{s\,r.m.s} = \frac{120+120\times0.1}{30}$ .....(01)

(for substitution)

$= 4.4$ A .....(01)

Part (B)

(a) Consider the AND gate circuit made from transistors operating as switches shown in figure (1). The circuit consists of three *npn* transistors T_1, T_2 and T_3. Inputs A and B control the action of the transistors T_1 and T_2, while transistor T_3 controls the final output X. The circuit operates from a $V_{CC} = +5$ V power supply. Assume that $V_{BE} = 0{\cdot}7$ V, $\beta = 100$, and V_{CE} at saturation is 0·2 V for all the transistors. Desired collector currents for T_1 and T_2 are 4 mA and that for T_3 is 4·8 mA.

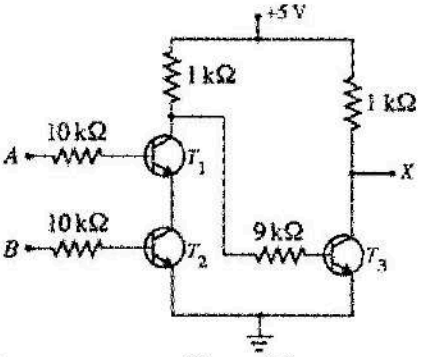


Figure (1)

(i) Consider the situation when both inputs A and B are at 5 V.

(I) Calculate the base current of T_2. Hence, show that T_2 is in saturation state.

(II) Calculate the base current of T_1. Hence, show that T_1 is in saturation state.

(ii) Consider the situation when either $A = 5$ V and $B = 0$ V or $A = 0$ V and $B = 5$ V. State the operational state (ON or OFF) of T_1 and T_2 for each considering the current conduction from collector to emitter. No calculations are needed.

(iii) Calculate the base current of T_3 when either T_1 or T_2 is OFF. Hence, show that T_3 is in saturation state.

(iv) What are the values of the output voltage V_X for the following input conditions? State the operating mode (either ON or OFF) of T_3 for each case.

Case 1: $A = 5$ V and $B = 5$ V

Case 2: $A = 5$ V and $B = 0$ V

Case 3: $A = 0$ V and $B = 0$ V

(*b*) Consider the block diagram shown in figure (2) of a logic comparator which compares two binary digits A_1 and B_1. The output F_1 becomes 1 only if A_1 and B_1 are the same.

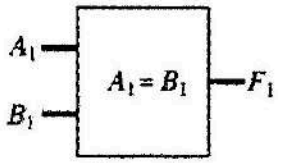


Figure (2)

(i) Write down the truth table of the comparator.

(ii) Write down the logic expression of the comparator using the above truth table.

(iii) Write down the truth table and logic expression of an XOR gate with inputs A_1 and B_1. Use it to write down a logic expression for the comparator.

(iv) Draw the logic circuit of the comparator using an XOR gate and a NOT gate.

(v) Draw the logic circuit of the comparator using only XOR gates.
Hint: Connect one input of an XOR gate permanently to logic 1 or 0 as needed.

(vi) Using the block diagram shown in figure (2) above and one additional 3-input logic gate, draw the composite diagram for a 3-bit comparator comparing A_1 and B_1, A_2 and B_2, A_3 and B_3.

(*c*) Consider logic gates *P* and *Q* of two different types. The logic voltage levels of the inputs and outputs are given in the table.

Logic Gate	Input		Output	
	Logic 1	Logic 0	Logic 1	Logic 0
P	2 V to 5 V	0 V to 0·8 V	2·7 V to 5 V	0 V to 0·4 V
Q	3·5 V to 5 V	0 V to 1·5 V	4·95 V to 5 V	0 V to 0·05 V

Logic gates of types *P* and *Q* are used to construct a logic circuit.

(i) In one circuit, the output of *P* is connected to the input of *Q*. Do you expect the circuit to operate properly? Briefly explain.

(ii) In another circuit, the output of *Q* is connected to the input of *P*. Do you expect the circuit to operate properly? Briefly explain.

(*a*) (i) I. $I_{B2} = \frac{(5-0.7)}{10\times10^3}$ .....(01)

$= 430\ \mu A\ (4.3 \times 10^{-4}\ A)$ .....(01)

The required base current for saturation $I_C/\beta = 4/100 = 40\ \mu A$

Since $I_{B2} > I_C/\beta$ *OR* $430\ \mu A > 40\ \mu A$ .....(01)
the base current is sufficient to operate T_2 in saturation.

II. $I_{B1} = \frac{(5-0.7-0.2)}{10\times10^3}$ .....(01)

$= 410\ \mu A\ (4.1 \times 10^{-4}\ A)$ .....(01)

The required base current for saturation $I_C/\beta = 4/100 = 40\ \mu A$

Since $I_{B1} > I_C/\beta$ *OR* $410\ \mu A > 40\ \mu A$ .....(01)
the base current is sufficient to operate T_1 in saturation

(ii) For $A = 5$ V and $B = 0$ V ; T_2 is OFF, T_1 is OFF .....(01)

For $A = 0$ V and $B = 5$ V ; T_1 is OFF, T_2 is OFF .....(01)

(iii) $I_{B3} = \frac{(5-0.7)}{(9+1)\times 10^3}$ .....(01)

$= 430\ \mu A\ (4.3 \times 10^{-4}\ A)$ .....(01)

The required base current for saturation $I_C/\beta = 4.8/100 = 48\ \mu A$
Since $I_{B3} > I_C/\beta$ *OR* $430\ \mu A > 48\ \mu A$ .....(01)
the base current is sufficient to operate T_3 in saturation.

(iv) Case 1: $A = 5$ V and $B = 5$ V

Case 2: $A = 5$ V and $B = 0$ V

Case 3: $A = 0$ V and $B = 0$ V

Case 1: T_3 is OFF and $V_X = 5$ V .....(01)

Case 2: T_3 is ON and $V_X = 0.2$ V .....(01)

Case 3: T_3 is ON and $V_X = 0.2$ V .....(01)

(*b*) (i)

A_1	B_1	F_1
0	0	1
0	1	0
1	0	0
1	1	1

.....(02)

(02 *or* 00)

(ii) $F_1 = (\bar{A}_1\bar{B}_1 + A_1B_1)$ .....(02)
(01 mark for each correct term)

(iii)

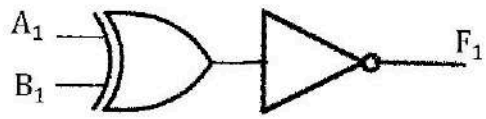
A_1	B_1	F_{XOR}
0	0	0
0	1	1
1	0	1
1	1	0

.....**(02)**

(02 *or* 00)

$F_1 = \overline{A_1 XOR B_1}$ *OR* $F_1 = \overline{A_1 \oplus B_1}$ *OR* $F_1 = \overline{A_1}B_1 + A_1\overline{B_1}$ .....**(01)**

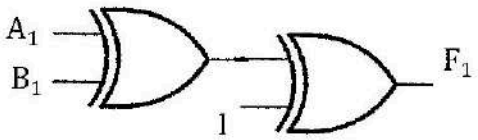
(iv)



..... **(02)**

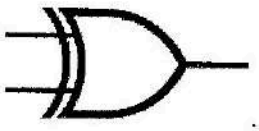
(02 *or* 00)

(v)

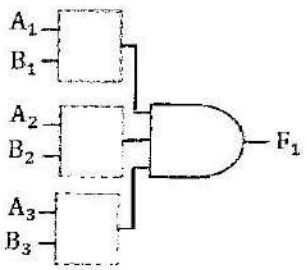


.....**(02)**

(02 *or* 00)

[XOR gates can be drawn as ]

(vi)



.....**(01)**

(*c*)

(i) No .....**(01)**

Logic high output voltage levels of *P* are outside of logic high input voltage levels of *Q* .....**(01)**

(ii) Yes .....**(01)**

Logic output voltage levels of *Q* are inside/within the logic input voltage levels of *P* .....**(01)**

10. Answer either part (A) or part (B) only.

Part (A)

(*a*) The first law of thermodynamics for a closed system can be written as $\Delta Q = \Delta U + \Delta W$. Clearly identify each term.

(*b*) What do you mean by an isothermal process, a constant pressure (isobaric) process and an adiabatic process?

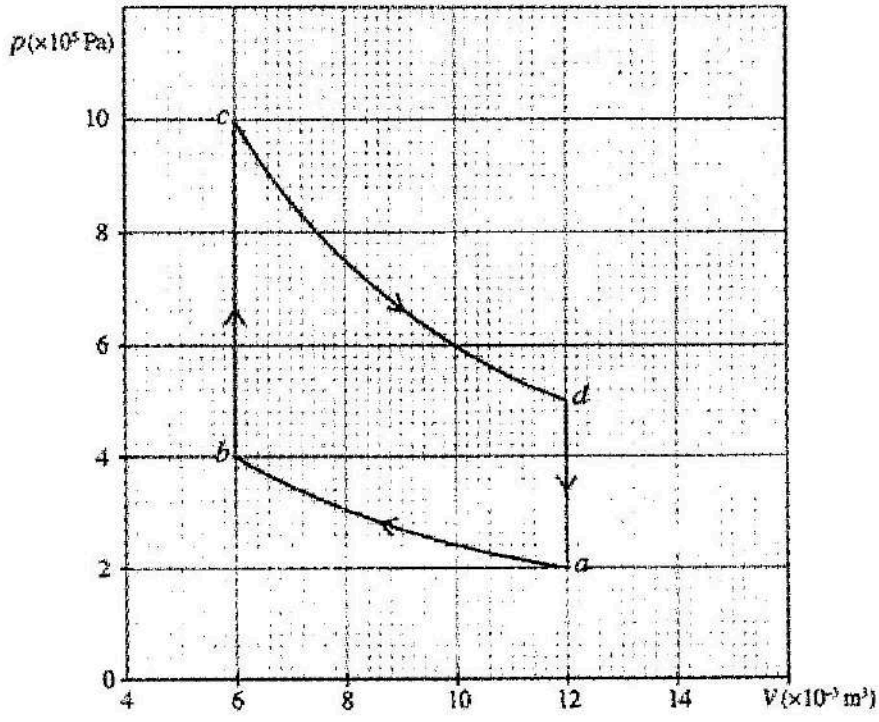
(*c*) Starting from the same point and labelling it as *A*, sketch the above three processes in the same p–V diagram. Mark isothermal, constant pressure and adiabatic processes as *AX*, *AY* and *AZ* respectively.

(i) Which process does obey Boyle's law?

(ii) Which process does obey Charles's law?

(iii) If the volume is increased from V_1 to V_2 at a pressure P_1 in an isobaric process write down an expression for ΔW in terms of P_1, V_1 and V_2.

(*d*) The Stirling engine converts heat to mechanical energy and that was invented by Robert Stirling in 1816. It is operated by a cyclic process of a closed system of an ideal gas by exposing it to different temperatures. A typical Stirling cycle is shown in the given p–V diagram by *abcda* cyclic path.



(i) Giving reasons identify the types of **four** processes *ab*, *bc*, *cd* and *da*.

(ii) If the temperature at point *a* is 273 °C, find temperatures at points *b*, *c* and *d*.

(iii) For a process represented by a vertical line such as *bc* the change in internal energy is given by the equation $\Delta U_{bc} = \frac{3}{2}(P_c - P_b)V_b$ where P_b and P_c are pressure at points *b* and *c* respectively. The volume at *b* is V_b. Calculate the heat energy supplied to the system in processes *bc* and *da*.

(iv) Assuming *ab* and *cd* are straight lines for calculation purpose only, find the work done in processes *ab* and *cd*.

(v) Using the same assumption in (*d*) (iv) above calculate the net work done in one cycle.

(vi) Using the same assumption in (*d*) (iv) above calculate the efficiency of the cyclic process *abcda*?

(*a*) ΔQ = Heat supplied to the system (No marks for change in heat) .....**(01)**

ΔU = Increase (or change) in internal energy .....**(01)**

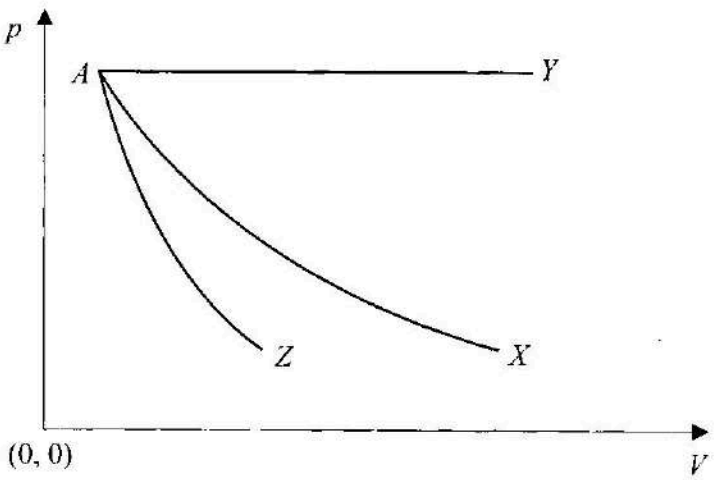
ΔW = Work done by the system (No marks for change in work done) .....**(01)**

(*b*) Isothermal process:
The temperature of the system remains constant/same throughout the process (or $\Delta T = 0$)
.....**(01)**

Isobaric process (constant pressure):
The pressure of the system remains constant/same throughout the process (or $\Delta p = 0$)
.....**(01)**

Adiabatic process:
The amount of heat in the system remains constant throughout the process. *OR* No heat gain or loss from the system throughout the process *OR* $\Delta Q = 0$)
.....**(01)**

(*c*)



[01 mark for each line with correct labelling. Do not look for (0,0)]

.....**(03)**

(i)

Boyle's law: *AX OR* isothermal process .....**(01)**

Charles's law: *AY OR* constant pressure process .....**(01)**

(ii) $\Delta W = p(V_2 - V_1)$ .....**(02)**

(*d*) (i) For *ab* $12\times2 = 6\times4$ and for *cd* $12\times5 = 6\times10$. Since $p_1V_1 = p_2V_2$ .....(01)
Therefore, *ab* and *cd* are isothermal ($\Delta T = 0$) processes .....(01)

$V_b = V_c$ and $V_d = V_a$ *OR* vertical lines .....(01)
Therefore, *bc* and *da* are constant volume (isochoric) ($\Delta V = 0$) processes.....(01)

(ii) Since *ab* is an isothermal ($\Delta T = 0$) process,
temperature at *b* is 273 °C. .....(01)

Since *bc* is an isochoric ($\Delta V = 0$) process $\frac{p_1}{T_1} = \frac{p_2}{T_2}$
and temperature at *c* is 1092 °C. .....(01)

Since *cd* is an isothermal ($\Delta T = 0$) process,
temperature at *d* is 1092 °C. .....(01)

(iii) Since $\Delta W_{bc} = 0$, ...(01)

$\Delta Q_{bc} = \Delta U_{bc}$

$\Delta Q_{bc} = \frac{3}{2}(10-4)\times10^5\times6\times10^{-3}$ .....(01)
(for correct substitution)

$= +5400$ J .....(01)

Since $\Delta W_{da} = 0$, $\Delta Q_{da} = \Delta U_{da}$

$\Delta Q_{da} = \frac{3}{2}(2-5)\times10^5\times12\times10^{-3}$

$= -5400$ J .....(01)
(to earn this mark – sign is essential)

(iv) Work done in *ab* = area under *ab* with right to left is (-).

$\Delta W_{ab} = \frac{(4+2)\times10^5}{2}\times(6-12)\times10^{-3}$

$= -1800$ J .....(01)
(to earn this mark – sign is essential)

Work done in *cd* = area under *cd* with left to right is (+).

$\Delta W_{cd} = \frac{(10+5)\times10^5}{2}\times(12-6)\times10^{-3}$

$= +4500$ J .....(01)

(v) $\Delta W = \Delta W_{ab} + \Delta W_{bc} + \Delta W_{cd} + \Delta W_{da}$

Since $\Delta W_{bc} = 0$ and $\Delta W_{da} = 0$

$\Delta W = -1800 + 0 + 4500 + 0$ .....(01)
(for subtraction)

$= +2700$ J .....(01)

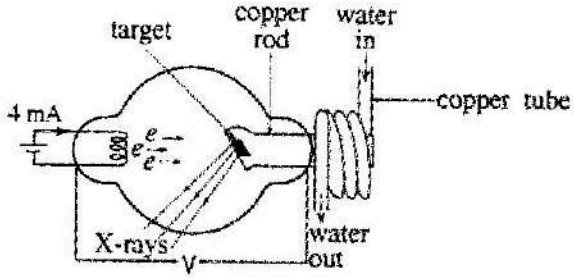
(vi) $\text{Efficiency} = \frac{\text{work done}}{\text{heat absorbed}} \times 100\%$

$\text{Efficiency} = \frac{2700}{4500} \times 100\%$ .....(01)
(for substitution)

$= 60\%$ .....(01)

Part (B)

(a) The figure shows a schematic diagram of an X-ray tube. It operates at $V = 30$ kV and the filament current is 4 mA.



(i) Determine the number of electrons (n) hitting the target per second. Electron charge $= 1{\cdot}6 \times 10^{-19}$ C.

(ii) Calculate the total kinetic energy (K) of all electrons hitting the target per second. Assume that the initial kinetic energy of electrons emitting from the filament is negligible.

(iii) Out of the energy calculated in (a) (ii) above, 95% is converted into heat in the target metal. This generated heat is taken out using a copper rod covered with a spiral copper tube connected to running water. Calculate the mass rate m (in kg min^{-1}) of flow of water if the temperature increase of water is 57 °C. Take the specific heat capacity of water as 4000 J kg^{-1} °C^{-1}.

(b) (i) Calculate the minimum wavelength (λ_{min}) of the emitted X-rays. Planck constant $h = 6{\cdot}6 \times 10^{-34}$ J s and speed of light $c = 3{\cdot}0 \times 10^{8}$ m s^{-1}.

(ii) Does the λ_{min} value calculated above depend on the target material? Give reasons for your answer.

(iii) If the filament current increases does the λ_{min} value calculated above change? Give reasons for your answer.

(iv) The target metals are usually made of tungsten or molybdenum. What are the reasons for this?

(c) (i) X-ray beam of intensity 5×10^3 W m^{-2} is incident on a human organ with an effective surface area of 0·01 m^2. Calculate the total energy delivered on to the organ in one second.

(ii) If the mass of the organ is 0·5 kg, calculate the absorbed dose in Gray (1 Gy = 1 J kg^{-1}).

(iii) What is the best material that can be used to effectively block or shield X-rays?

(iv) (I) Why is it important to measure the effective absorbed dose of radiation (in Sv) for people working in radiation environment?

(II) What might be the reason that the effective absorbed dose vary between different types of radiation, even when the absorbed dose is the same?

(d) When an atom is struck by a high-energy electron an inner electron can be ejected creating a vacancy in the inner energy level. An outer electron can transit to this vacancy emitting a photon with energy equal to the difference between energy levels. This process can generate X-rays with definite frequency. If the energies of the higher and lower levels are E_1 and E_2 respectively, the frequency f of the emitted X-ray photon is given by $hf = E_1 - E_2$. Here h is the Planck constant.

(i) For aluminium if $E_1 = -74$ eV and $E_2 = -1624$ eV, calculate the energy (in eV) of the X-ray photon emitted when an electron transition occurs from higher energy level to lower energy level.

(ii) Determine the corresponding wavelength of the X-ray photon produced. Take $hc = 1240$ eV nm.

(e) How do hard X-rays and soft X-rays differ from each other in terms of energy, wavelength, and penetrating power.

(a) (i) $I = ne$

$$n = \frac{4\times10^{-3}}{1.6\times10^{-19}}$$

$$= 2.5 \times 10^{16}$$

(ii) KE of a single electron = eV .....(01)

Total KE of all electrons = neV

$$= 2.5 \times 10^{16} \times 1.6 \times 10^{-19} \times 30 \times 10^3$$

$$= 120 \text{ J s}^{-1} \text{ (W)}$$

(iii) $120 \times \frac{95}{100} = m' \times 4000 \times 57$ .....(02)

(01 mark for taking 95% ; 01 mark for R.H.S)

$m = m' \times 60$ .....(01)

(for multiplying by 60)

$= 0.03$ kg min^{-1}

(*b*) (i) $\lambda_{min} = \frac{hc}{eV}$ .....(02)

$= \frac{6.6\times10^{-34}\times3.0\times10^{8}}{1.6\times10^{-19}\times30\times10^{3}}$ .....(01)

(for correct substitution)

$= 4.125 \times 10^{-11}$ m $[(4.12 - 4.13) \times 10^{-11}$ m] .....(01)

(ii) No .....(01)

Minimum wavelength occurs for electrons stop at once upon colliding with the target *OR* when electrons loses all of its kinetic energy in a single collision. Therefore λ_{min} does not depend on the target material. .....(01)

(iii) No .....(01)

Increase of filament current does not change kinetic energy of electrons. (It merely increases the number of electrons hitting the target) .....(01)

(iv) They are chosen because they have high melting point and higher atomic number.

.....(02)

(01 mark for each)

(*c*) (i) Total energy $= 5 \times 10^3 \times 0.01$

$= 50$ J .....(01)

(ii) Absorbed dose $= \frac{50}{0.5}$ .....(01)

(for the division)

$= 100$ Gy .....(01)

(iii) Lead/tungsten/tin/bismuth/antimony *OR* concrete .....(02)

(iv) I. The effective dose depends on both the absorbed dose and the biological impact of different types of radiation. .....(01)

II. The RBE factor/ quality factor/Q factor may have different values depending on the radiation type. .....(01)

(*d*) (i) Energy of the X ray $= -74 - (-1560)$ .....(01)

$= 1486$ eV .....(02)

(ii) $\lambda = \frac{hc}{E}$ .....(01)

$\lambda = \frac{1240}{1486}$ .....(01)

(for substitution)

$= 0.84$ nm $(8.4 \times 10^{-10}$ m) .....(01)

(*e*) **Energy**: Hard X-rays have higher energy than soft X-rays. .....(01)

Wavelength: Hard X-rays have shorter wavelengths whereas soft X-rays have longer wavelengths *OR* Hard X-rays have shorter wavelengths compared to soft X-rays *OR* vice versa

.....(01)

Penetrating Power: Hard X-rays can penetrate deeper into materials *OR* penetration power of hard X-rays is higher compared to soft X-rays *OR* vice versa

.....(01)